BIPOLAR ANTI-FUZZY SUB RINGS

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Abstract— In this paper, we define a new algebraic structure of a bipolar anti-fuzzy sub ring and investigate some of its related properties. The purpose of this study is to implement the fuzzy set theory and ring theory in bipolar anti-fuzzy subring of a ring.

Keywords— fuzzy ring, anti-fuzzy ring, fuzzy subring, anti-fuzzy subring, bipolar fuzzy set, bipolar fuzzy ring, bipolar anti-fuzzy ring.

I. INTRODUCTION

The concept of fuzzy sets was initiated by Zadeh [10]. Then it has become a vigorous area of research in engineering, medical science, social science, graph theory etc. In 1982 Wang-jin Liu [3] introduced the concept of fuzzy ring and fuzzy ideal. Liu defined the fuzzy ideals of a ring and discussed the operations on fuzzy ideals. In fuzzy sets the membership degree of elements range over the interval [0,1]. The membership degree expresses the degree of belongingness of elements to a fuzzy set. The membership degree 1 indicates that an element completely belongs to its corresponding fuzzy set and membership degree 0 indicates that an element does not belong to fuzzy set. The membership degrees on the interval (0, 1) indicate the partial membership to the fuzzy set. Sometimes, the membership degree means the satisfaction degree of elements to some property or constraint corresponding to a fuzzy set. The author W.R.Zhang [8],[9] commence the concept of bipolar fuzzy sets as a generalization of fuzzy sets in 1994. In case of Bipolar-valued fuzzy sets membership degree range is enlarged from the interval [0, 1] to [-1, 1]. In a bipolar-valued fuzzy set, the membership degree 0 means that the elements are irrelevant to the corresponding property, the membership degree (0,1] indicates that elements somewhat satisfy the property and the membership degree [-1,0) indicates that elements somewhat satisfy the implicit counter-property.

In this paper we introduce the concept of bipolar anti-fuzzy subrings and discuss some of its properties.

II. PRELIMINARIES

In this section, we sit the fundamental definitions that will be used in the sequel. Throughout this paper, R = (R ,+, ·) is a Ring, e is the additive identity element of R and xy, we mean x · y.

Definition 2.1[7]
A fuzzy set μ on R is said to be a fuzzy subring on R, if for every x,y ∈ R,

i. μ (x−y) ≥ min { μ (x) , μ (y)}
ii. μ (xy) ≥ min { μ (x) , μ (y)}.

Definition 2.2[5]
A fuzzy set μ on R is said to be an anti-fuzzy subring on R, if for every x,y ∈ R,

i. μ (x−y) ≤ max { μ (x) , μ (y)}
ii. μ (xy) ≤ max { μ (x) , μ (y)}.

Definition 2.3[4,5]
Let X be a non-empty set. A bipolar-valued fuzzy (BVF) set or bipolar fuzzy set B in X is an object having the form B = {⟨x, μB+ (x), μB− (x)⟩ / for all x ∈ X},where μB+ : X → [0,1] and μB− : X → [-1,0] are mappings. The positive membership
degree $\mu_B^+(x)$ denotes the satisfaction degree of an element $x$ to the property corresponding to a bipolar-valued fuzzy set $B = \{(x, \mu_B^+(x), \mu_B^-(x)) \mid \forall x \in X\}$ and the negative membership degree $\mu_B^-(x)$ denotes the satisfaction degree of an element $x$ to some implicit counter property corresponding to a bipolar-valued fuzzy set $B = \{(x, \mu_B^-(x), \mu_B^+(x)) \mid \forall x \in X\}$. For the sake of simplicity, we shall use the symbol $B = (\mu_B^+, \mu_B^-)$ for the bipolar-valued fuzzy set $B = \{(x, \mu_B^+(x), \mu_B^-(x)) \mid \forall x \in X\}$.

**Definition 2.4[6,7]**

Let $x = (\lambda_1, \lambda_2)$ and $\mu = (\mu_1^+, \mu_1^-)$ be two bipolar fuzzy sets in $R$ then the anti product of $(\lambda \times \mu) : R \times R \rightarrow [0,1]$ is defined by $(\lambda \times \mu ^-)(x,y) = \max \{\lambda_1^-(x), \mu_1^-(y)\}$ and $(\lambda \times \mu ^+)(R \times R \rightarrow [-1,0]$ is defined by $(\lambda \times \mu ^+)(x,y) = \min \{\lambda_1^+(x), \mu_1^+(y)\}$.

**III. PROPERTIES OF BIPOLAR ANTI-FUZZY SUBRINGS**

In this section, we introduce the bipolar anti-fuzzy subring and discuss some of its properties.

**Definition 3.1[8]**

A bipolar fuzzy set $\mu$ is called a bipolar fuzzy subring of $R$ if $\mu$ satisfies i. $\mu^*(x-y) \geq \min \{\mu^*(x), \mu^-(y)\}$,

ii. $\mu^-(x-y) \leq \max \{\mu^-(x), \mu^-(y)\}$,

iii. $\mu(x+y) \geq \min \{\mu^+(x), \mu_0^+(y)\}$,

iv. $\mu(x+y) \leq \max \{\mu^-(x), \mu^-(y)\}$.

**Definition 3.2**

A bipolar fuzzy set $\mu$ is called a bipolar anti-fuzzy subring of $R$ if $\mu$ satisfies

i. $\mu^*(x-y) \leq \max \{\mu^*(x), \mu^+(y)\}$,

ii. $\mu^-(x-y) \geq \min \{\mu^-(x), \mu^-(y)\}$,

iii. $\mu(x+y) \leq \max \{\mu^+(x), \mu_0^+(y)\}$,

iv. $\mu(x+y) \geq \min \{\mu^-(x), \mu^-(y)\}$.

**Theorem 3.3**

Let $B_1 = (\mu_{B_1}^+, \mu_{B_1}^-)$ and $B_2 = (\mu_{B_2}^+, \mu_{B_2}^-)$ be two bipolar anti-fuzzy subrings of $R$ then $B_1 \cap B_2$ is also a bipolar anti-fuzzy subring of $R$.

**Proof:** Let $B_1 = (\mu_{B_1}^+, \mu_{B_1}^-)$ and $B_2 = (\mu_{B_2}^+, \mu_{B_2}^-)$ be two bipolar anti-fuzzy subrings of $R$. Let $x, y \in R$ then

$$(\mu_{B_1} \cap B_2) (x - y) = \max \{ (\mu_{B_1}^+) (x - y), (\mu_{B_2}^+) (x - y) \}$$

$$\leq \max \{ \max \{ (\mu_{B_1}^+) (x), (\mu_{B_1}^-) (y) \}, \max \{ (\mu_{B_2}^+) (x), (\mu_{B_2}^-) (y) \} \}$$

$$\leq \max \{ \max \{ (\mu_{B_1}^+) (x), (\mu_{B_2}^+) (y) \}, \max \{ (\mu_{B_1}^-) (x), (\mu_{B_2}^-) (y) \} \}$$

$$\leq \max \{ (\mu_{B_1} \cap B_2)^+ (x), (\mu_{B_1} \cap B_2)^- (y) \}$$

also

$$(\mu_{B_1} \cap B_2) (x - y) = \min \{ (\mu_{B_1}^-) (x - y), (\mu_2^-) (x - y) \}$$

$$\geq \min \{ \min \{ (\mu_{B_1}^-) (x), (\mu_{B_1}^-) (y) \}, \min \{ (\mu_{B_2}^-) (x), (\mu_{B_2}^-) (y) \} \}$$

$$\geq \min \{ \min \{ (\mu_{B_1}^-) (x), (\mu_{B_2}^-) (x) \}, \min \{ (\mu_{B_1}^-) (y), (\mu_{B_2}^-) (y) \} \}$$

$$\geq \min \{ (\mu_{B_1} \cap B_2)^- (x), (\mu_{B_1} \cap B_2)^- (y) \}$$

now

$$(\mu_{B_1} \cap B_2) (xy) = \max \{ (\mu_{B_1}^+) (xy), (\mu_{B_2}^+) (xy) \}$$

$$\leq \max \{ \max \{ (\mu_{B_1}^+) (x), (\mu_{B_1}^+) (y) \}, \max \{ (\mu_{B_2}^+) (x), (\mu_{B_2}^+) (y) \} \}$$

$$\leq \max \{ \max \{ (\mu_{B_1}^+) (x), (\mu_{B_2}^+) (x) \}, \max \{ (\mu_{B_1}^+) (y), (\mu_{B_2}^+) (y) \} \}$$

$$\leq \max \{ (\mu_{B_1} \cap B_2)^+ (x), (\mu_{B_1} \cap B_2)^+ (y) \}$$

also

$$(\mu_{B_1} \cap B_2) (xy) = \min \{ (\mu_{B_1}^-) (xy), (\mu_{B_2}^-) (xy) \}$$

$$\geq \min \{ \min \{ (\mu_{B_1}^-) (x), (\mu_{B_1}^-) (y) \}, \min \{ (\mu_{B_2}^-) (x), (\mu_{B_2}^-) (y) \} \}$$

$$\geq \min \{ \min \{ (\mu_{B_1}^-) (x), (\mu_{B_2}^-) (x) \}, \min \{ (\mu_{B_1}^-) (y), (\mu_{B_2}^-) (y) \} \}$$

$$\geq \min \{ (\mu_{B_1} \cap B_2)^- (x), (\mu_{B_1} \cap B_2)^- (y) \}$$
\[
\begin{align*}
\geq \min\{\min\{\{(\mu_{B_1})^-(x), (\mu_{B_2})^-(x)\}, \min\{\{(\mu_{B_1})^-(y), (\mu_{B_2})^-(y)\}\}
\geq \min\{\{(\mu_{B_1} \cap B_2)^-(x), (\mu_{B_1} \cap B_2)^-(y)\}\}
\end{align*}
\]

Hence \( B_1 \cap B_2 \) is a bipolar anti-fuzzy subring of \( R \).

**Theorem 3.4**

Let \( B_1 = (\mu_{B_1}^+, \mu_{B_1}^-) \) and \( B_2 = (\mu_{B_2}^+, \mu_{B_2}^-) \) be two bipolar anti-fuzzy subrings of \( R \) then \( B_1 \times B_2 \) is also a bipolar anti-fuzzy subring of \( R \times R \).

**Proof:** Let \( B_1 = (\mu_{B_1}^+, \mu_{B_1}^-) \) and \( B_2 = (\mu_{B_2}^+, \mu_{B_2}^-) \) be two bipolar anti-fuzzy subrings of \( R \). Let \( x_1 \) and \( x_2 \) be in \( R \), \( y_1 \) and \( y_2 \) be in \( R \). Then \((x_1, y_1)\) and \((x_2, y_2)\) are in \( R \times R \).

Now,
\[
(\mu_{B_1} \times B_2)(x_1, y_1) - (x_2, y_2) = (\mu_{B_1} \times B_2)(x_1 - x_2, y_1 - y_2)
\]
\[
= \max\{\max\{(\mu_{B_1})(x_1), (\mu_{B_2})(y_1)\}, \max\{(\mu_{B_1})(x_2), (\mu_{B_2})(y_2)\}\}
\]
\[
\leq \max\{\max\{(\mu_{B_1})(x_1), (\mu_{B_1})(x_2)\}, \max\{(\mu_{B_2})(y_1), (\mu_{B_2})(y_2)\}\}
\]
\[
= \max\{\min\{(\mu_{B_1})(x_1, y_1), (\mu_{B_1})(x_2, y_2)\}\}
\]

Therefore,
\[
(\mu_{B_1} \times B_2)(x_1, y_1) - (x_2, y_2) \leq \min\{\{(\mu_{B_1} \times B_2)(x_1, y_1)\}, \{(\mu_{B_1} \times B_2)(x_2, y_2)\}\}
\]

also
\[
(\mu_{B_1} \times B_2)(x_1, y_1) - (x_2, y_2) = \min\{\{(\mu_{B_1})(x_1), (\mu_{B_2})(y_1)\}, \{(\mu_{B_1})(x_2), (\mu_{B_2})(y_2)\}\}
\]
\[
\geq \min\{\min\{(\mu_{B_1})(x_1, y_1), (\mu_{B_1})(x_2, y_2)\}\}, \min\{\min\{(\mu_{B_2})(x_1), (\mu_{B_2})(y_1)\}, \min\{\min\{(\mu_{B_2})(x_2), (\mu_{B_2})(y_2)\}\}\}
\]
\[
= \min\{\min\{(\mu_{B_1})(x_1, y_1), (\mu_{B_1})(x_2, y_2)\}\}
\]

Therefore,
\[
(\mu_{B_1} \times B_2)(x_1, y_1) - (x_2, y_2) \geq \min\{\{(\mu_{B_1} \times B_2)(x_1, y_1)\}, \{(\mu_{B_1} \times B_2)(x_2, y_2)\}\}
\]

and,
\[
(\mu_{B_1} \times B_2)(x_1, y_1) - (x_2, y_2) \geq \min\{\{(\mu_{B_1})(x_1, y_1)\}, \{(\mu_{B_1})(x_2, y_2)\}\}
\]

Theorem 3.5

Let \( \lambda \) and \( \mu \) be bipolar fuzzy subsets of the rings \( R_1 \) and \( R_2 \) respectively. Suppose that \( e_1 \) and \( e_2 \) are the identity elements of \( R_1 \) and \( R_2 \) respectively. If \( \mu \times \varphi \) is a bipolar anti-fuzzy subring of \( R_1 \times R_2 \), then at least one of the following two
statements must hold.

i. \( \mu^+(e_2) \leq \lambda^+(x), \mu^-(e_2) \geq \lambda^-(x) \), for all \( x \) in \( F_1 \)

ii. \( \lambda^+(e_1) \leq \mu^+(y), \lambda^-(e_1) \geq \mu^-(y) \), for all \( y \) in \( F_2 \).

**Proof:** Let \( \lambda \times \mu \) be a bipolar anti-fuzzy subring of \( R_1 \times R_2 \). By contraposition, suppose that none of the statements (i) and (ii) holds. Then we can find \( x \) in \( F_1 \) and \( y \) in \( R_2 \) such that \( \mu^+(e_2) > \lambda^+(x), \mu^-(e_2) < \lambda^-(x) \) and \( \lambda^+(e_1) < \mu^+(y), \lambda^-(e_1) > \mu^-(y) \). We have

\[
(\lambda \times \mu)^+(x,y) = \max \{ \lambda^+(x), \mu^+(y) \} < \max \{ \mu^+(e_2), \lambda^+(e_1) \} = \max \{ \mu^+(e_2), \mu^+(e_2) \} = (\lambda \times \mu)^+(e_1, e_2)
\]

\[
(\lambda \times \mu)^-(x,y) < (\lambda \times \mu)^-(e_1, e_2)
\]

\[
(\lambda \times \mu)^+(x,y) = \min \{ \lambda^-(x), \mu^-(y) \} > \min \{ \mu^-(e_2), \lambda^-(e_1) \} = \min \{ \lambda^-(e_1), \mu^-(e_2) \} = (\lambda \times \mu)^-(e_1, e_2)
\]

Thus, \( \lambda \times \mu \) is not a bipolar anti-fuzzy subring of \( \Omega_1 \times \Omega_2 \).

Hence, either \( \mu^+(e_2) \leq \lambda^+(x), \mu^-(e_2) \geq \lambda^-(x) \), for all \( x \) in \( R_1 \)

or \( \lambda^+(e_1) \leq \mu^+(y), \lambda^-(e_1) \geq \mu^-(y) \), for all \( y \) in \( R_2 \).

**Theorem 3.6**

Let \( \lambda \) and \( \mu \) be bipolar fuzzy subsets of \( R_1 \) and \( R_2 \) respectively, such that \( \lambda^+(x) \geq \mu^+(e_2), \lambda^-(x) \leq \mu^-(e_2) \), for all \( x \) in \( F_1 \), \( \phi_2 \) is the identity element of \( F_2 \). If \( \lambda \times \mu \) is a bipolar anti-fuzzy subring of \( F_1 \times F_2 \), then \( \lambda \) is a bipolar anti-fuzzy subring of \( F_1 \).

**Proof:** It is clear.

**Theorem 3.7**

Let \( \lambda \) and \( \mu \) be bipolar fuzzy subsets of \( R_1 \) and \( R_2 \) respectively, such that \( \mu^+(e_2) \geq \lambda^+(e_1), \mu^-(e_2) \leq \lambda^-(e_1) \), for all \( x \) in \( F_2 \), \( \phi_1 \) is the identity element of \( F_1 \). If \( \lambda \times \mu \) is a bipolar anti-fuzzy subring of \( F_1 \times F_2 \), then \( \mu \) is a bipolar anti-fuzzy subring of \( F_2 \).

**Proof:** It is clear.

**Definition 3.8**

Let \( R \) be a ring and \( \phi \neq A \subseteq R \) then the bipolar valued fuzzy characteristic function \( \psi_A = (\psi_A^+, \psi_A^-) \) of \( A \) is defined as

\[
\psi_A^+ = \begin{cases} 
1 & \text{if } x \in A \\
0 & \text{if } x \notin A
\end{cases} \quad \text{and} \quad \psi_A^- = \begin{cases} 
-1 & \text{if } x \in A \\
0 & \text{if } x \notin A
\end{cases}
\]

**Theorem 3.9:**

Let \( A \) be a non-empty subset of \( R \) then \( A \) is an subring of \( R \) then \( \psi_A \) is a bipolar anti-fuzzy subring of \( R \).

**Proof:** Let \( A \) be a subring of \( R \). For any \( x, y \in R \). We have the following cases

**Case(i):** If \( x, y \in A \) then \( x - y \in A \) also \( xy \in A \). Since \( A \) is a subring of \( R \).

\[
\begin{align*}
\psi_A^+(x) &= 1 & \psi_A^+(y) &= 1 & \psi_A^+(x-y) &= 1 & \psi_A^+(xy) &= 1 \\
\psi_A^-(x) &= -1 & \psi_A^-(y) &= -1 & \psi_A^-(x-y) &= -1 & \psi_A^-(xy) &= -1
\end{align*}
\]

now \( \psi_A^+(x-y) \leq 1 \leq \max \{ 1, 1 \} \leq \max \{ \psi_A^+(x), \psi_A^+(y) \} \)

\[
\begin{align*}
\psi_A^+(x-y) &\leq 1 \\
\psi_A^-(x-y) &\geq -1
\end{align*}
\]

\[
\begin{align*}
\psi_A^-(x-y) &\geq 1 \\
\psi_A^-(x-y) &\leq 1
\end{align*}
\]
Case(iii): If \( x \in A \) or \( y \in A \) then

(i) if \( x \in A \), \( y \notin A \) then \( x - y \in A \) and \( xy \notin A \).

\[
\begin{align*}
\psi_A^+(x) &= 1 & \psi_A^-(x) &= -1 \\
\psi_A^+(y) &= 0 & \psi_A^-(y) &= 0 \\
\psi_A^+(x-y) &= 1 & \psi_A^-(x-y) &= -1
\end{align*}
\]

now, \( \psi_A^+(x-y) = -1 \leq \max \{1,1\} \leq \max \{ \psi_A^+(x), \psi_A^-(y) \} \)

\[
\begin{align*}
\psi_A^-(x-y) &= 0 & \psi_A^+(x-y) &= 1
\end{align*}
\]

and

\[
\begin{align*}
\psi_A^+(xy) &= 0 \leq \max \{0,0\} \leq \max \{ \psi_A^+(x), \psi_A^-(y) \} \\
\psi_A^-(xy) &= 0 \geq \min \{0,0\} \geq \min \{ \psi_A^-(x), \psi_A^+(y) \}
\end{align*}
\]

(ii) if \( x \notin A \), \( y \in A \) then \( x - y \notin A \) and \( xy \notin A \).

\[
\begin{align*}
\psi_A^+(x) &= 0 & \psi_A^-(x) &= 0 \\
\psi_A^+(y) &= 1 & \psi_A^-(y) &= 0 \\
\psi_A^+(x-y) &= 0 & \psi_A^-(x-y) &= 0
\end{align*}
\]

now, \( \psi_A^+(x-y) = 0 \leq \max \{0,1\} \leq \max \{ \psi_A^-(x), \psi_A^+(y) \} \)

\[
\begin{align*}
\psi_A^-(x-y) &= 0 & \psi_A^+(x-y) &= 1
\end{align*}
\]

and

\[
\begin{align*}
\psi_A^+(xy) &= 0 \leq \max \{0,1\} \leq \max \{ \psi_A^+(x), \psi_A^-(y) \} \\
\psi_A^-(xy) &= 0 \geq \min \{0,1\} \geq \min \{ \psi_A^-(x), \psi_A^+(y) \}
\end{align*}
\]

Therefore, \( \psi_A \) is a bipolar anti-fuzzy subring of \( R \).

IV. Conclusions

In this paper we introduced the new algebraic structure bipolar anti fuzzy HX ring successfully discussed with their some important properties on a HX ring defined on a finite HX ring.

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